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# MIXING AND 1-LOOP FLAVOR STRUCTURE OF FERMIONIC CURRENTS IN THE STANDARD MODEL OF ELECTROWEAK INTERACTIONS

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**Abstract:** We show that, unlike mass matrices, the fermionic gauge currents of the Standard Model exhibit, at the quantum level, remarkable $SU(2)_f$ flavor properties at the observed values of the mixing angles. They accommodate all measured mixing for three families of quarks, and, for neutrinos, maximal $\theta_{23}$, quark-lepton complementarity $\tan 2\theta_c = 1/2 \leftrightarrow \tan 2\theta_{12} = 2$, and a not so small $\sin^2 2\theta_{13} = .267$ within the present $90\%$ $c.l.$ interval of the $T2K$ experiment.

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## 1 Introduction

We wish to combine two fairly old concepts which have long been proven fruitful: Current Algebra [1] and Gell-Mann's use of "strong $SU(3)$" symmetry [2]):

* much physics is conveyed by *currents* and is strongly constrained by their algebraic properties;
* the laws of transformations of terms in a Lagrangian which *break* a given symmetry carry important physical information; this is how Gell-Mann deduced his famous mass formulæ for hadrons.

The currents under concern here are the gauge currents of the Standard Model of electroweak interactions, and the symmetry group flavor $SU(2)_f$ and its $U(1)$ subgroup of flavor rotations between pairs of fermions carrying the same electric charge.

## 2 The Standard Model in flavor space for two generations of quarks

### 2.1 The classical Lagrangian

The basics are more easily explained with two generations of quarks only, $(u,d)$ and $(c,s)$. These are the usual $SU(2)_L$ doublets, that we immediately reshuffle into the two flavor doublets $(u_f, c_f)$ and $(d_f, s_f)$. We use the subscript "$_f$" to denote (bare) flavor eigenstates. No transition exists at this order between $c_f$ and $u_f$, nor between $s_f$ and $d_f$: these states are (classically) orthogonal.

It is convenient to embed the (global part of the) electroweak group into the chiral flavor group $U(4)_L \times U(4)_R$. The electroweak $SU(2)_L$ generators write then at the classical level ($\mathbb{1}$ is the $2\times 2$ unit matrix)

$$T^3 = \frac{1}{2}\left(\begin{array}{c|c} \mathbb{1} & \\ \hline & -\mathbb{1} \end{array}\right), T^+ = \left(\begin{array}{c|c} & \mathbb{1} \\ \hline & \end{array}\right), T^- = \left(\begin{array}{c|c} & \\ \hline \mathbb{1} & \end{array}\right). \tag{1}$$

Gauge currents form an $SU(2)_L$ triplet: they are obtained by sandwiching the generators in eq. (1) between $((u,c),(d,s))$ fermions and inserting the appropriate $\gamma$ matrices. Neutral currents are controlled

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by $T^3$; they form two singlets of $SU(2)_f$: classically there is no flavor changing neutral current (FCNC) (non-diagonal terms are vanishing) nor any violation of universality (diagonal terms are two by two identical).

The standard Cabibbo phenomenology makes use of Yukawa couplings to induce two mass matrices for the fermions, one for $(u_f, c_f)$, one for $(d_f, s_f)$. After they are diagonalized, which introduces two mixing angles $\theta_u$ and $\theta_d$ the $SU(2)_L$ generators write in the space of mass eigenstates $(u_m, c_m), (d_m, s_m)$

$$T^3 = \frac{1}{2}\left(\begin{array}{c|c} 1 & \\ \hline & -1 \end{array}\right), T^+ = \left(\begin{array}{c|c} & C \\ \hline & \end{array}\right), T^- = \left(\begin{array}{c|c} & \\ \hline C^\dagger & \end{array}\right), \tag{2}$$

in which $C$ is the unitary Cabibbo matrix $C = \left(\begin{array}{cc} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{array}\right), \quad \theta_c = \theta_u - \theta_d$.

## 2.2 Its symmetries

Let $\mathcal{R}(\omega) = \left(\begin{array}{cc} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{array}\right)$. The Cabibbo matrix $C$ and the fermion masses stay unchanged if one makes arbitrary and independent flavor rotations $\left(\begin{array}{c} u_f \\ c_f \end{array}\right) \to \mathcal{R}(\theta)\left(\begin{array}{c} u_f \\ c_f \end{array}\right), \left(\begin{array}{c} d_f \\ s_f \end{array}\right) \to \mathcal{R}(\phi)\left(\begin{array}{c} d_f \\ s_f \end{array}\right)$.

However, preserving $SU(2)_L$ symmetry requires that fermions belonging to the same $SU(2)_L$ doublet be transformed with the same phase; thus $\theta$ must be equal to $\phi$, which leaves a $U(1)$ invariance.

The Cabibbo angle $\theta_c$ stays arbitrary, also corresponding to an arbitrary $U(1)$ rotation. This symmetry (arbitrariness) gets broken in nature, but there is no hint at the classical level of how it is broken.

## 2.3 Notations

It is convenient for the following to introduce the set of three flavor $SU(2)_f$ generators, which depend on a (mixing) angle $\alpha$:

$$\mathcal{T}_x(\alpha) = \frac{1}{2}\left(\begin{array}{cc} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{array}\right), \quad \mathcal{T}_y = \frac{1}{2}\left(\begin{array}{cc} & -i \\ i & \end{array}\right), \quad \mathcal{T}_z(\alpha) = \frac{1}{2}\left(\begin{array}{cc} \sin 2\alpha & -\cos 2\alpha \\ -\cos 2\alpha & -\sin 2\alpha \end{array}\right). \tag{3}$$

A 2-dimensional flavor rotation also writes $\mathcal{R}(\alpha) = e^{2i\alpha\mathcal{T}_y}$ and $\mathcal{T}_{x,z}(\alpha)$ satisfy the relation

$$\mathcal{R}^\dagger(\omega)\,\mathcal{T}_{x,z}(\alpha)\,\mathcal{R}(\omega) = \mathcal{T}_{x,z}(\alpha+\omega). \tag{4}$$

## 2.4 At 1-loop

$s_m \to d_m, c_m \to u_m$ transitions spoil the orthogonality of bare flavor states and Cabibbo phenomenology [3], unless one introduces a series of counterterms [4]. They are both kinetic and mass-like, with both chiralities. In the $(d,s)$ sector, they write (the expressions are analogous in the $(u,c)$ sector) [5]

$$-A_d\,\bar d_m\,\not p(1-\gamma^5)\,s_m - B_d\,\bar d_m(1-\gamma^5)s_m - E_d\,\bar d_m\,\not p(1+\gamma^5)\,s_m - D_d\,\bar d_m(1+\gamma^5)s_m, \tag{5}$$

with

$$A_d = \frac{m_d^2 f_d(p^2=m_d^2) - m_s^2 f_d(p^2=m_s^2)}{m_d^2 - m_s^2}, \qquad E_d = \frac{m_s m_d \left(f_d(p^2=m_d^2) - f_d(p^2=m_s^2)\right)}{m_d^2 - m_s^2},$$
$$B_d = -m_s E_d, \qquad D_d = -m_d E_d, \tag{6}$$

In the unitary gauge for the $W$ boson and dimensionally regularized

$$f_d = g^2 \sin\theta_c \cos\theta_c (m_c^2 - m_u^2) \int_0^1 dx \left[\frac{2x(1-x)}{\Delta(p^2)} + \frac{p^2 x^3 (1-x)}{M_W^2 \Delta(p^2)} + \frac{x + 3x^2}{M_W^2 \Delta(p^2)^{2-n/2}} \Gamma(2-n/2)\right]. \tag{7}$$

$n = 4 - \epsilon$ is the dimension of space-time, $\Delta(p^2) = (1-x)M_W^2 + x\frac{m_u^2+m_c^2}{2} - x(1-x)p^2$, and $\Gamma$ is the Euler Gamma function which satisfies in particular $\Gamma(\epsilon/2) = 2/\epsilon - \gamma + \ldots$ where $\gamma \approx 0.5772\ldots$ is the Euler constant. We shall come back later to more specific properties.

### 2.5 1-loop mixing and currents

Adding counterterms to the bare Lagrangian modifies its diagonalization and thus mixing matrices. We define the mixing matrix $\mathcal{C}_d$ in the left-handed $(d,s)$ sector by $\begin{pmatrix} d_f \\ s_f \end{pmatrix} = \mathcal{C}_d \begin{pmatrix} d_m \\ s_m \end{pmatrix}$, in which $d_m, s_m$ are the new mass eigenstates (similar expressions hold in the $(u,c)$ sector).

Finding the new expressions of gauge currents is simple [5] thanks to a straightforward consequence of $SU(2)_L$ gauge invariance which dictates that the derivatives in the kinetic-like counterterms be replaced by the $SU(2)_L$ covariant derivatives. In a first step one shows that they are controlled by the unit matrix in the new mass basis. Stepping back to the original bare flavor basis, one finds [5] that they are controlled by the $2\times 2$ matrices $(\mathcal{C}_d^{-1})^\dagger \mathcal{C}_d^{-1} = 1 + 2A_d \mathcal{T}_z(\theta_d)$ in the $(d,s)$ sector, and $(\mathcal{C}_u^{-1})^\dagger \mathcal{C}_u^{-1} = 1 + 2A_u \mathcal{T}_z(\theta_u)$ in the $(u,c)$ sector. These expressions exhibit in particular the non-unitarity of the mixing matrices $\mathcal{C}_{u,d}$ connecting bare flavor states to 1-loop mass states, which has already been extensively commented upon [6][5][7][8].

Likewise, the charged current writes [5] $\begin{pmatrix} \bar u_f & \bar c_f \end{pmatrix} [1 + A_u \mathcal{T}_z(\theta_u) + A_d \mathcal{T}_z(\theta_d)]\, \gamma_L^\mu \begin{pmatrix} d_f \\ s_f \end{pmatrix}$, which is also controlled, in the bare flavor basis, by a non-unitary matrix [1].

The back-reaction of introducing 1-loop counterterms is accordingly that the classical mixing angles $\theta_u$ and $\theta_d$ now occur inside the gauge currents expressed in the bare flavor basis. Some formal structure starts to appear through the $SU(2)_f$ generator $\mathcal{T}_z$. It is also important that only the $A$ counterterms occur, because of the renormalization freedom attached to them, that we shall soon focus on.

### 2.6 Flavor rotations

Like in the classical case, we perform flavor rotations, respectively with angles $\theta$ on $(u_f, c_f)$ and $\phi$ on $(d_f, s_f)$. Since the classical Cabibbo angle $\theta_c$ is unchanged, one easily shows that the counterterms also stay identical. The neat effect of these arbitrary transformations is finally the one obtained naively by transforming the neutral and charged currents given above and using the relations (4). The neutral currents get now controlled in bare flavor space by $1 + 2A_d \mathcal{T}_z(\theta_d + \phi)$ in the $(d,s)$ sector, and $1 + 2A_u \mathcal{T}_z(\theta_u + \theta)$ in the $(u,c)$ sector. The charged current becomes (omitting the fermions and the $\gamma$ matrix) $1 + A_u \mathcal{T}_z(\theta_u + (1/2)(\theta+\phi)) + A_d \mathcal{T}_z(\theta_d + (1/2)(\theta+\phi))$. Like before, going through all the steps, one can show that the Cabibbo angle stays unchanged, whatever $\theta$ and $\phi$, and so do the mass eigenvalues. So, "physics

[1] It is important to recall that the mixing matrices $\mathfrak{C}_d, \mathfrak{C}_d$ and $\mathfrak{C}$ which connect 1-loop mass eigenstates to 1-loop flavor states (instead of bare flavor states) are unitary and satisfy the relation $\mathfrak{C} = \mathfrak{C}_u^\dagger \mathfrak{C}_d$ [5]. The standard Cabibbo (CKM) phenomenology is indeed restored at 1-loop by the counterterms.

is unchanged". Arguing with $SU(2)_L$ symmetry to impose $\phi = \theta$, the arguments of all $\mathcal{T}_z$ generators become $\theta_{u,d} + \phi$. The arbitrariness of $\phi$ reflects an up-to-now unbroken $U(1)$ invariance.

At 1-loop, neutral currents are no longer flavor singlets of $SU(2)_f$: there exist, in bare flavor space, both FCNC's and violations of universality (differences among diagonal terms); *flavor rotations continuously transform FCNC's into violation of universality and vice-versa*.

### 2.7 Left-over freedom

Since an arbitrary rotation by $\phi$ does not change "physics", we can choose $\phi = -\theta_u$, which aligns classical flavor and mass $(u, c)$ fermions.

The counterterms $A_d, B_d, D_d, E_d$ satisfy the conditions $B_d = cst$ (thus $E_d = cst$ and $E_d = cst$ and there is no freedom on $B, E, D$). $A_u$ and $A_d$ have both a pole in $n - 4 = \epsilon$ but the combination $(m_c^2 - m_u^2)A_u - (m_s^2 - m_d^2)A_d$ is finite [2]. This shows in particular that at least one non unitary mixing matrices occur in one of the two channels $(u, c)$ and $(d, s)$. One has the freedom to renormalize to $0$ one among the two counterterms $A_u$ or $A_d$. Let us choose $A_u = 0$; the alignment of flavor and mass states that we imposed classically by choosing $\phi = -\theta_u$ is maintained at 1-loop. Then, $A_d$ can only be non-vanishing $A_d \approx -g^2 \sin^2\theta_c \frac{27}{12}\left(\frac{m_c}{m_W}\right)^4 \approx 1.24\,10^{-7}$.

After these manipulations, the neutral currents are respectively controlled by $1$ in the $(u, c)$ channel, $1 + 2A_d\mathcal{T}_z(\theta_d - \theta_u)$ in the $(d, s)$ channel, and the charged currents by $1 + A_d\mathcal{T}_z(\theta_d - \theta_u)$, such that they all only depend on the arbitrary Cabibbo angle through the $\mathcal{T}_z(\theta_c)$ generator.

### 2.8 How flavor rotations get broken

Once all freedom has been used, the arbitrariness of $\phi$ has become that of the Cabibbo angle.

We noticed that classical neutral gauge currents only project on $SU(2)$ flavor singlets. The simplest extension is that quantum corrections add to it a part proportional to a triplet of $SU(2)_f$, that is

$$\begin{pmatrix} \bar d & \bar s \end{pmatrix} \left[ \begin{pmatrix} 0 & \pm 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ \pm 1 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \gamma^\mu_L \begin{pmatrix} d \\ s \end{pmatrix},$$

which corresponds in the Lagrangian to $\begin{pmatrix} \bar d & \bar s \end{pmatrix} \begin{pmatrix} 1/2 & \pm 1 \\ \pm 1 & -1/2 \end{pmatrix} \gamma^\mu_L \begin{pmatrix} d \\ s \end{pmatrix}$ (the three components of the triplet are coupled to the same gauge boson, unlike what happens for the $SU(2)_L$ triplet of electroweak currents (1)).

Comparing with the expressions obtained in subsection 2.5 and matching the term proportional to $\mathcal{T}_z$ [3] with the formulæ above, one gets the relation $\tan 2\theta_c = \pm 1/2$, which corresponds with very good accuracy to the measured value of the Cabibbo angle. The choices that we made in order to align at 1-loop $(u, c)$ flavor and mass states ensure that the same angle that occurs in the $(d, s)$ neutral current also occurs inside charged currents, which get similar structures [4].

### 2.9 The last generator: $\mathcal{T}_x$ and the mass matrix

Of the three $SU(2)_f$ generators, we have up to now only used $\mathcal{T}_y$, which, by exponentiation, triggers flavor rotations, and $\mathcal{T}_z$ which controls gauge currents. It is natural to look for $\mathcal{T}_x$ in the mass terms since it is the third element of the process: diagonalization $\to$ mixing matrix $\to$ gauge currents. And, indeed,

[2] It is $g^2 \sin^2\theta_c \frac{(m_c^2 - m_u^2)(m_s^2 - m_d^2)}{m_W^4}\left[\frac{27}{12}(m_c^2 + m_u^2) + \frac{23}{24}(m_s^2 + m_d^2)\right]\left(1 + \mathcal{O}(m^2/m_W^2)\right)$.

[3] The general expression of $\mathcal{T}_z$ is given in eq. (3).

[4] The $U(1)$ breaking occurs in such a way that the violation of universality $1/2 - (-1/2) = 1$ is identical to the "amount" of FCNC which is also 1. So, flavor rotations, which, as we stressed, continuously transform the former into the latter, are broken in such a way that the two violations occur with the same strength.

supposing, to simplify, that the mass matrix is symmetric, it writes

$M = \left(\begin{array}{cc} a & c \\ c & b \end{array}\right) = \frac{m_2+m_1}{2} + (m_2 - m_1)\mathcal{T}_x(\theta)$, where $m_1$ and $m_2$ are the eigenvalues of $M$ and $\tan 2\theta = \frac{2c}{a-b}$. This corresponds to the decomposition of the symmetric non-degenerate matrix $M$: $M = m_1 P_1 + m_2 P_2$, where $P_1 = \frac{1}{2}(1 - 2\mathcal{T}_x(\theta))$ and $P_2 = \frac{1}{2}(1 + 2\mathcal{T}_x(\theta))$ are the projectors on the sub-spaces of eigenvectors of $M$.

So, the whole set $\{$masses, currents$\}$ of the Standard Model is controlled by the $SU(2)_f$ flavor group with generators given by (3).

Most efforts have been devoted to establishing connections between the mass spectrum $(m_1, m_2)$ and the Cabibbo angle (see for example [9]). This is probably not the appropriate way to proceed for at least two reasons:

* any homographic transformation on a mass matrix $M : M \rightarrow \frac{\alpha M+\beta}{\delta M+\gamma}$ leaves unchanged the eigenvectors of $M$ and thus the mixing angles: an infinite number of different mass matrices, with different eigenvalues, therefore correspond to the same mixing angle;

* the structure that we found within gauge currents stays hidden at the level of the mass matrix. Indeed, the $(d,s)$ mass matrix corresponding to $\tan 2\theta_d = \frac{1}{2}$ is $M = \frac{m_s+m_d}{2} + \frac{m_s-m_d}{\sqrt{5}}\left(\begin{array}{cc} 1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{array}\right) = \left(\begin{array}{cc} m_s(\frac{1}{2}+\frac{1}{\sqrt{5}}) + m_d(\frac{1}{2}-\frac{1}{\sqrt{5}}) & \frac{m_s-m_d}{2\sqrt{5}} \\ \frac{m_s-m_d}{2\sqrt{5}} & m_s(\frac{1}{2}-\frac{1}{\sqrt{5}}) + m_d(\frac{1}{2}+\frac{1}{\sqrt{5}}) \end{array}\right)$. Though its part proportional to $(m_s - m_d)$ is obtained from the triplet part of neutral currents by a rotation $\theta \rightarrow \theta + \frac{\pi}{4}$, it has as a whole no conspicuous property.

## 3 Three generations of quarks

The simplest generalization of what has been done for two generations is to consider, inside each flavor triplet of quarks, $(u,c,t)$ and $(d,s,b)$, the three $2\times 2$ flavor rotations associated with the three quark pairs, and the corresponding three sets of four currents, for example $[\bar d\gamma^\mu_L s, \bar s\gamma^\mu_L d, \bar s\gamma^\mu_L s, \bar d\gamma^\mu_L d]$, $[\bar d\gamma^\mu_L b, \bar b\gamma^\mu_L d, \bar b\gamma^\mu_L b, \bar d\gamma^\mu_L d]$ and $[\bar b\gamma^\mu_L s, \bar s\gamma^\mu_L b, \bar s\gamma^\mu_L s, \bar b\gamma^\mu_L b]$. One then requests that each set decomposes into a trivial singlet part + a triplet of the corresponding $SU(2)_f$. This is akin to requesting that the matrix $(\mathcal{C}_d^{-1})^\dagger \mathcal{C}_d^{-1}$ which controls neutral currents, be of the form [5] [5]

$$(\mathcal{C}_d^{-1})^\dagger \mathcal{C}_d^{-1} = 1 + \left(\begin{array}{ccc} \alpha & \pm(\alpha-\beta) & \pm(\alpha-\gamma) \\ \pm(\alpha-\beta) & \beta & \pm(\beta-\gamma) \\ \pm(\alpha-\gamma) & \pm(\beta-\gamma) & \gamma \end{array}\right). \tag{8}$$

Decomposing the $3\times 3$ mixing matrix $\mathcal{C}_d$ as a product of three "quasi-rotations" acting in each of the three 2-dimensional flavor sub-spaces, and introducing, like in the $2\times 2$ case, two mixing angles for each

[5] Eq. (8) also writes $(\mathcal{C}_d^{-1})^\dagger \mathcal{C}_d^{-1} =$

$1 + (\alpha-\beta)\left(\begin{array}{ccc} 1/2 & \pm 1 & 0 \\ \pm 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right) + (\beta-\gamma)\left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1/2 & \pm 1 \\ 0 & \pm 1 & -1/2 \end{array}\right) + (\alpha-\gamma)\left(\begin{array}{ccc} 1/2 & 0 & \pm 1 \\ 0 & 0 & 0 \\ \pm 1 & 0 & -1/2 \end{array}\right)$
$+ \frac{\alpha}{2}\left(\begin{array}{ccc} 0 & & \\ & 1 & \\ & & 1 \end{array}\right) + \frac{\beta}{2}\left(\begin{array}{ccc} 1 & & \\ & 0 & \\ & & 1 \end{array}\right) + \frac{\gamma}{2}\left(\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 0 \end{array}\right)$, making explicit its decomposition on $SU(2)_f$ singlets and triplets.

of them, this yields a set of trigonometric equations which have been investigated and solved in [5]. Even if the whole set of solutions was not produced, it was shown that it includes values of the mixing angles in remarkable agreement with observation.

This accordingly supports the conjecture that $2 \times 2$ flavor rotations get spontaneously broken such that the corresponding neutral currents decompose onto a singlet + a (small) triplet component of $SU(2)_f$. At this point it is useful to note that the decomposition (8) is more restrictive than a singlet + octet of $SU(3)_f$, because the Gell-Mann matrices $\lambda_2, \lambda_5$ and $\lambda_7$ do not appear.

It is still more hopeless than for two generations to look for any particular structure in the mass matrix.

# 4 Neutrinos

## 4.1 Where do they take their maximal mixing from?

• The perturbative vacuum of the Standard Model is unstable while the true vacuum, away from which one can safely quantize small variations, cannot be reached perturbatively from the former. Let us transpose this type of consideration to mixing angles. The usual "classical solution" corresponds here to a unitary $\mathcal{C}_d = \mathcal{R}(\theta_d)$ mixing matrix with an arbitrary $\theta_d$. We call it "Cabibbo-like". Can there be other classical solutions? If we parametrize the mixing matrix by a rotation $\mathcal{R}(\theta_d)$, we have no chance whatsoever to eventually find them. We should instead parametrize it in a general non-unitary form, since we know that this is how it is ultimately realized, then find all possible solutions at the "classical limit" $\mathcal{C}_d^\dagger \mathcal{C}_d \to 1$. This has been done in [6] where we parametrized (for two generations)
$\mathcal{C}_d = \left(\begin{array}{cc} e^{i\alpha} c_1 & e^{i\delta} s_1 \\ -e^{i\beta} s_2 & e^{i\gamma} c_2 \end{array}\right)$, $c_1 = \cos\theta_1, s_2 = \sin\theta_2\ etc.$ The first type of solutions are "Cabibbo-like" and correspond to
* $\theta_2 = \theta_1 + k\pi$ associated with $e^{i(\alpha-\delta)} = e^{i(\beta-\gamma)}$,
or to
* $\theta_2 = -\theta_1 + k\pi$ associated with $e^{i(\alpha-\delta)} = -e^{i(\beta-\gamma)}$;
the second type of solutions are sets of discrete values [6]
* $[\theta_1 = (k-n)\frac{\pi}{2}, \theta_2 = (k+n)\frac{\pi}{2}]$ or $[\theta_1 = \frac{\pi}{4} + (n-k)\frac{\pi}{2}, \theta_2 = \frac{\pi}{4} + (n+k)\frac{\pi}{2}]$, both associated with $e^{i(\alpha-\delta)} = e^{i(\beta-\gamma)}$,
* $[\theta_1 = (n-k)\frac{\pi}{2}, \theta_2 = (n+k)\frac{\pi}{2}]$ or $[\theta_1 = -\frac{\pi}{4} + (k-n)\frac{\pi}{2}, \theta_2 = \frac{\pi}{4} + (k+n)\frac{\pi}{2}]$, both associated with $e^{i(\alpha-\delta)} = -e^{i(\beta-\gamma)}$,
which include in particular the maximal mixing $\pm\frac{\pi}{4}$.

• In the case of three generations, the measured values of the neutrino mixing angles point at two "Cabibbo-like" classical mixing angles $\theta_{12}, \theta_{13}$, combined with a maximal $\theta_{23}$. It is again simple to see how maximal mixing springs out there. Let us do this in the case where the third mixing angle $\theta_{13}$ is, for the sake of simplicity, taken to vanish. One can even take a parametrization of the mixing matrix without the phases that were introduced in the case of two generations [5]:
$\mathcal{C}^{-1} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -\tilde{s}_{23} & \tilde{c}_{23} \end{array}\right) \left(\begin{array}{ccc} c_{12} & s_{12} & 0 \\ -\tilde{s}_{12} & \tilde{c}_{12} & 0 \\ 0 & 0 & 1 \end{array}\right)$. It gives the following expression for the symmetric matrix $(\mathcal{C}^{-1})^\dagger \mathcal{C}^{-1}$ which controls neutral currents:

$$(\mathcal{C}^{-1})^\dagger \mathcal{C}^{-1} = \left(\begin{array}{ccc} c_{12}^2 + \tilde{s}_{12}^2(c_{23}^2 + \tilde{s}_{23}^2) & c_{12}s_{12} - \tilde{c}_{12}\tilde{s}_{12}(c_{23}^2 + \tilde{s}_{23}^2) & -\tilde{s}_{12}(c_{23}s_{23} - \tilde{c}_{23}\tilde{s}_{23}) \\ c_{12}^2 + \tilde{s}_{12}^2(c_{23}^2 + \tilde{s}_{23}^2) & s_{12}^2 + \tilde{c}_{12}^2(c_{23}^2 + \tilde{s}_{23}^2) & -\tilde{c}_{12}(c_{23}s_{23} - \tilde{c}_{23}\tilde{s}_{23}) \\ -\tilde{s}_{12}(c_{23}s_{23} - \tilde{c}_{23}\tilde{s}_{23}) & -\tilde{c}_{12}(c_{23}s_{23} - \tilde{c}_{23}\tilde{s}_{23}) & s_{23}^2 + \tilde{c}_{23}^2 \end{array}\right),$$

[6] which still satisfy $\theta_2 = \pm\theta_1 + k\pi$ and are thus superposed to the continuous sets of Cabibbo-like solutions.

where we used the abbreviated notations $\tilde{s}_{12} = \sin\tilde{\theta}_{12}$ etc . The "classical equations", which correspond to unitarity, are now five. Three constrain the non-diagonal elements to vanish and two express the identity of diagonal elements. Excluding $\tilde{\theta}_{12} = 0$, the vanishing of the $[1,3]$ and $[2,3]$ entries of $(\mathcal{C}^{-1})^\dagger \mathcal{C}^{-1}$ requires $\sin 2\theta_{23} = \sin 2\tilde{\theta}_{23}$, that is: either $\tilde{\theta}_{23} = \theta_{23} + k\pi$ or $\tilde{\theta}_{23} = \frac{\pi}{2} - \theta_{23} + k\pi$. Choosing the second possibility yields, for example $\tilde{s}_{23} = c_{23}$. Then, that the $[1,2]$ entry vanishes requires $c_{12}s_{12} = 2c_{23}^2\tilde{s}_{12}\tilde{c}_{12}$. That the entries $[1,1]$ and $[2,2]$ are identical writes $c_{12}^2 - s_{12}^2 = 2c_{23}^2(\tilde{c}_{12}^2 - \tilde{s}_{12}^2)$, and, last, that $[2,2]$ = $[3,3]$ requires $s_{12}^2 + 2c_{23}^2\tilde{c}_{12}^2 - 2s_{23}^2$. These last three equations let us note (a), (b) and (c). Making the ratio (a)/(b) yields $\tan 2\theta_{12} = \tan 2\tilde{\theta}_{12}$, which requires $\tilde{\theta}_{12} = \theta_{12} + \frac{k\pi}{2} + n\pi$. The same equations then yield $2c_{23}^2 = 1$, that is, $\theta_{23}$ *maximal*. It remains to verify that all equations are satisfied. They are indeed with $\tilde{\theta}_{23} = \frac{\pi}{2} - \theta_{23} + k\pi$ and $\tilde{\theta}_{12} = \theta_{12} + p\pi$. So, the classical solution corresponds here to a Cabibbo-like $\theta_{12}$, to maximal $\theta_{23}$, and to a (Cabibbo-like) $\theta_{13} = 0$ (which is our simplifying hypothesis). True values of the mixing angles are then sought for as solutions of eq. (8) restricted to lie in the vicinity of this set.

### 4.2 Mixing angles

Solving the corresponding system of trigonometric equations in full generality is quite an involved task but it is not our goal. We can instead keep the value of $\theta_{23}$ maximal as it has been obtained in the "classical" (unitary) limit, and solve in this limit the equations for $\theta_{12}$ and $\theta_{13}$ given by eq. (8). It is also convenient to start with $\theta_{13} = 0$. With these two constraints, one obtains [10][5] for $\theta_{12}$ the "golden ratio value" [10][11] $\tan 2\theta_{12} = 2$, which, associated with $\tan 2\theta_c = 1/2$ ensures the so-called "quarks-leptons complementarity" [12]. Imposing then this value for $\theta_{12}$ and still keeping $\theta_{23}$ maximal, one solves the equations for $\theta_{13}$ coming from eq. (8) in the approximation that it is small ($\sin^2\theta_{13} < .1$). This gives [5] two possible values $\sin^2 2\theta_{13} = 1.3\,10^{-4}$ and $\sin^2 2\theta_{13} = .267$. They lie within the recent $90\%\ c.l.$ interval of the $T2K$ experiment $\sin^2 2\theta_{13} < .5$ [13]. In this framework accordingly perfectly accommodate a value of $\theta_{13}$ not so small as it had long been expected.

## 5 Conclusion

We have used 1-loop counterterms to motivate explicit calculations of mixing angles at this order. However the results that we have obtained are general and only rely on two properties:

* Quantum corrections always induce non-unitarity for the mixing matrices connecting classical flavor eigenstates to 1-loop mass eigenstates of non-degenerate coupled fermions [6] [5] [8];

* Flavor rotations get broken in such a way that quantum corrections induce $SU(2)_f$ triplets in 1-loop neutral currents (expressed in terms of classical flavor eigenstates) while they only involve singlets at the classical level.

Among open issues one can mention:

* Why do, at the classical (unitary) limit, quarks exhibit three continuous "Cabibbo-like" mixing angles while neutrinos get a discrete value for one of them? We have seen that, if one pays enough attention, all these choices are perfectly legitimate (and also $\pm\frac{\pi}{2}$ values), but we have no dynamical explanation for them. In particular, why does, seemingly, only $\theta_{23}$ get a finite "classical value" ?

* The system of trigonometric equations resulting from the symmetry constraints that we impose has many solutions. That this set includes the subset of observed values gives no hint concerning why this subset is chosen by nature and why other allowed solutions are not.

* Why does $SU(2)$ seems to play again an important role? Is it more than an "opportunistic" (low-energy, at the best) symmetry only introduced because it is the simplest that comes to mind? Should it be given a more fundamental status like that of a (spontaneously broken or not) gauge symmetry?

Both the non-unitarity of mixing matrices in bare flavor space and the generation of $SU(2)_f$ triplets in neutral currents, which seemingly control mixing angles, originate from the existence of mass splittings. However, there is no direct connection between masses and mixing angles: the gap stays wide open and the mass spectrum remains apart. Gell-Mann's achievement to find relations among hadron masses from a

specific breaking pattern of $SU(3)$ $(u, d, s)$ flavor symmetry still waits for its equivalent for fundamental fermions. There lies most probably the realm of physics beyond the standard model.

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## References

[1] *S.L. Adler & R.F. Dashen, "Current Algebras and Applications to Particle Physics", Frontiers in Physics, Benjamin (New York, Amsterdam) 1968*

[2] *M. Gell-Mann & Y. Nee'man, "The Eightfold Way", Frontiers in Physics, Benjamin (New York, Amsterdam) 1964.*

[3] *Q. Duret, B. Machet & M.I. Vysotsky, "Quark Lagrangian diagonalization versus non-diagonal kinetic terms", arXiv:0810.4449 [hep-ph], Mod. Phys. Lett. A 24 (2009) 273-275.*

[4] *E.P. Shabalin, "Electric dipole moment of the quark in a gauge theory with left handed-fermions", Yad. Fiz. 28 (1978) 151 [Sov. J. Nucl. Phys. 28, (1978) 75].*

[5] *Q. Duret, B. Machet & M.I. Vysotsky, "Mixing angles of quarks and leptons in Quantum Field Theory", arXiv:0805.4121 [hep-ph], Eur. Phys. J. C 61 (2009) 247-278.*

[6] *Q. Duret & B. Machet, "Mixing Angles and Non-Degenerate Coupled Systems of Particles", hep-ph/0606303, Phys. Lett. B 643 (2006) 303-310.*

[7] *B. Machet, V. Novikov, M.I. Vysotsky, "Binary systems of neutral mesons in Quantum Field Theory", hep-ph/0407268, Int. J. Mod. Phys. A 20 (2005) 5399-5452.*

[8] *V. Novikov, "Binary systems in QM and in QFT: CPT", hep-ph/0509126 (2005)*

[9] *C. Giunti & C.W. Kim, "Fundamentals of Neutrino Physics and Astrophysics", Oxford University Press, 2007.*

[10] *Q. Duret & B. Machet, "The emergence of the Cabibbo angle in non-degenerate coupled systems of fermions", hep-ph/0610148, Phys. Lett. B 642 (2006) 469-471.*

[11] *Y. Kajiyama, M. Raidal & A. Strumia, "The golden ratio prediction for the solar neutrino mixing", arXiv:0705.4559 [hep-ph], Phys. Rev D 76, 117301 (2007)*

[12] *M. Raidal, "Prediction* $\theta_c + \theta_{sol} = \frac{\pi}{4}$ *from flavor physics: a new evidence for grand unification?", hep-ph/0404046, Phys. Rev. Lett. 93 (2004) 161801.*

[13] *Mark Hartz, for the T2K Collaboration, "First Oscillation Results for the T2K Experiment", arXiv:1201.1846 [hep-ex] (2012).*